Measuring the Moon with *shadows*

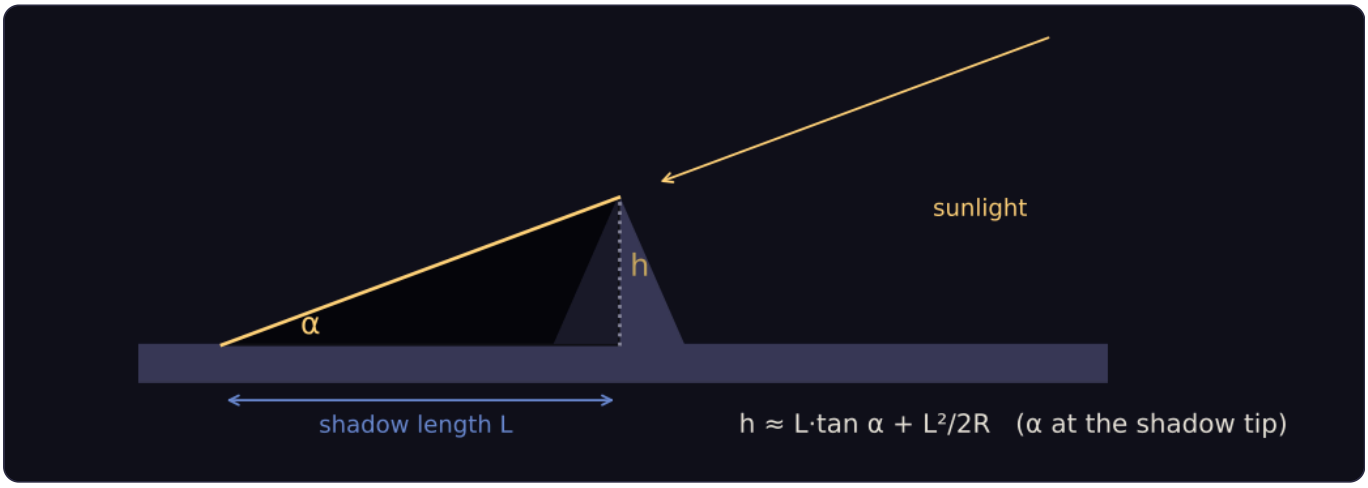
Two old methods applied to my own images. In 1610 Galileo saw that the Moon has mountains and worked out that they are several kilometres high. Today the heights are measured from the mountains' shadows. Aristarchus and Hipparchus used the Earth's shadow during a lunar eclipse to find the size and distance of the Moon. Both can be done today with a small telescope and a camera, even a phone held up to the eyepiece.

1 · Mountains and craters

When the Sun is low over a place on the Moon, mountains and crater rims cast long shadows. If we know the length L of a shadow and the altitude α of the Sun above the horizon at the tip of the shadow, the height is

$$h \approx L \cdot \tan \alpha$$

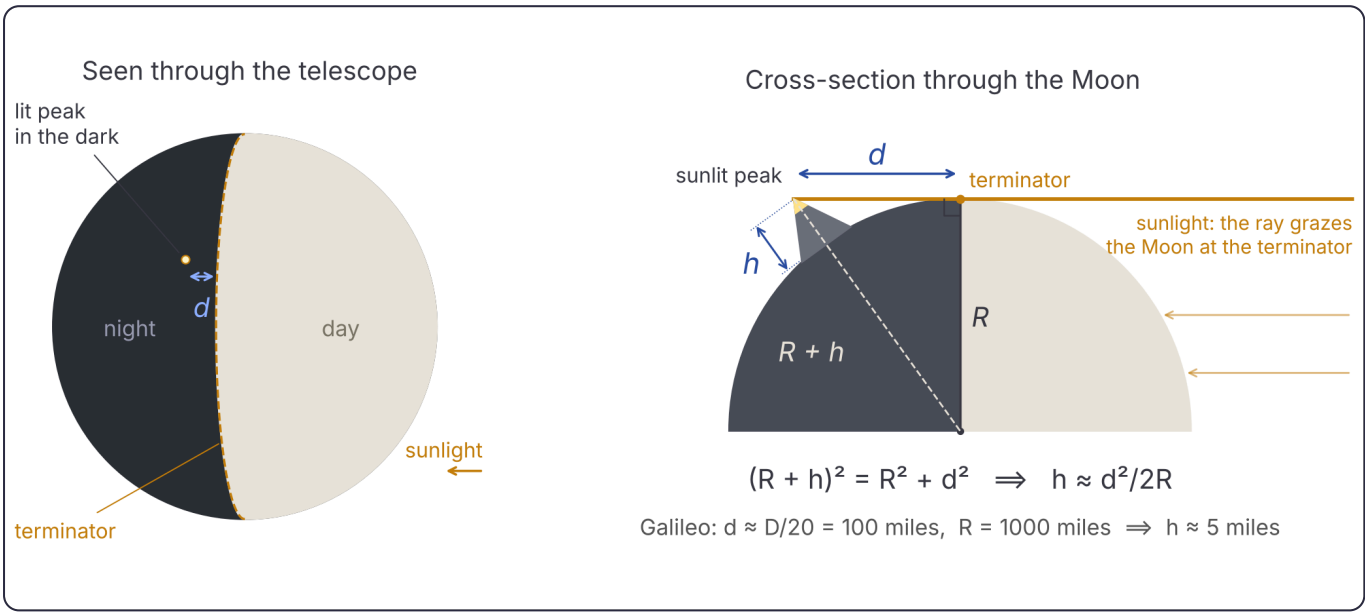
At very low solar altitudes the shadows are so long that the curvature of the Moon matters, so I work out the geometry fully in three dimensions: the sunlight grazes the summit and meets the surface at the tip of the shadow. To first order the curvature adds a term L²/(2R) when α is the solar altitude at the shadow tip. Using the solar altitude at the mountain instead, the term must be subtracted, because the Sun stands L/R higher there.



The shadow method. The solar altitude α is the angle between the sunlight and the local horizon.

GALILEO'S CALCULATION

Galileo did not measure the shadows themselves. In *Sidereus Nuncius* (1610) he describes bright points that appear in the dark part of the Moon some distance from the terminator. They are mountain tops already lit by the Sun while the ground around them is still dark. The ray of sunlight that just reaches the top grazes the lunar surface at the terminator. If the distance from the terminator to the top is d, Pythagoras gives $(R + h)^2 = R^2 + d^2$, so $h \approx d^2/2R$.



Galileo's method. Left: a mountain top in the dark part of the Moon already catches the sunlight at a distance d from the terminator. Right: the ray of sunlight that just reaches the top grazes the Moon at the terminator, so R, d and $R + h$ form a right-angled triangle. The height of the mountain is greatly exaggerated.

Galileo took the Moon's diameter to be 2000 Italian miles and saw bright tops up to 1/20 of the diameter, that is 100 miles, into the dark side. That gives $1000^2 + 100^2 = 1\,010\,000$, whose square root is just under 1005, so the top lies about 5 miles above the sphere. Galileo cautiously wrote "more than four miles", corresponding to 6–7 km, and concluded that the mountains of the Moon are higher than those of the Earth. It was an attack on the Aristotelian idea of a perfectly smooth celestial sphere. The figure is a little too large, but the order of magnitude is right, and the term $d^2/2R$ is the same curvature term that appears in the shadow formula above. Later astronomers measured the length of the shadows instead, and in the 1830s Beer and Mädler used them to find the heights of more than a thousand lunar mountains.

HEIGHT ABOVE WHAT?

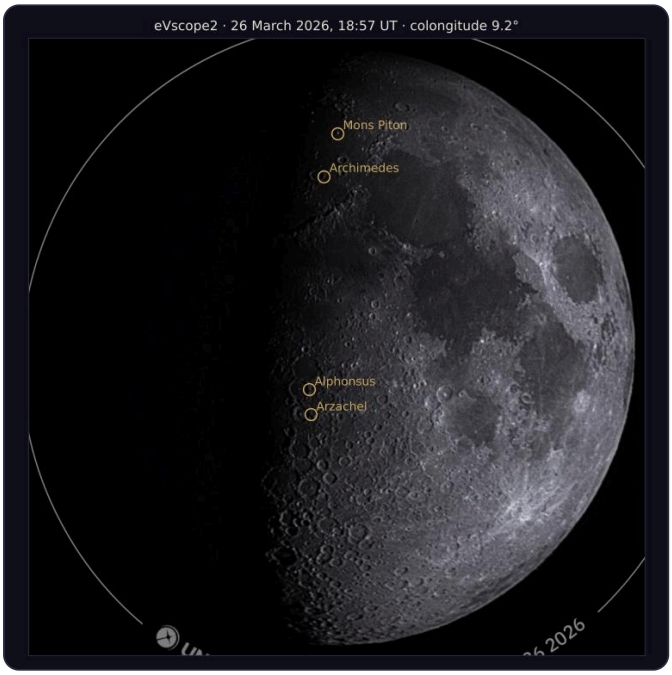
The Moon has no sea, so there is no sea level to measure from. Galileo's zero was implicitly the ideal sphere: he assumed that the ground at the terminator lay on the surface of the sphere. Today absolute heights are given above a reference sphere of radius 1737.4 km centred on the Moon's centre of mass. That is why the floor of Copernicus has a

height of about −3.4 km in NASA's elevation data. The highest point on the Moon lies almost 11 km above the sphere and the lowest just over 9 km below it, so the total range is about 20 km. The shadow method, by contrast, measures a relative height: the top above the place where the shadow ends. The published values for isolated mountains are also relative, but measured from surrounding terrain chosen by each author, and this explains some of the differences in the table below.

THE IMAGES

I used two of my eVscope2 images (built-in camera and stacking) with long shadows near the terminator: 26 March 2026 at 18:57 UT and 25 May 2026 at 22:01 UT (00:01 CEST on 26 May). The eVscope2 has a focal length of 450 mm and 2.9 µm pixels, so the scale is 1.329″ per pixel, or 2.4–2.5 km per pixel at the centre of the disc. As a check, the Moon in a raw frame from 22 November 2026 has a radius of 741.5 pixels, and the ephemeris gives 737–741 pixels for that evening.

For each image I found the centre and radius of the Moon from the bright limb, and the rotation of the image from 10 known craters. The fit is good to 2 pixels on average. From the time of exposure I calculated the position of the Sun as seen from the Moon, using the IAU rotation model for the Moon and an ephemeris for the Sun and Moon. That gives the Sun's colongitude, and from it the solar altitude at every single point.



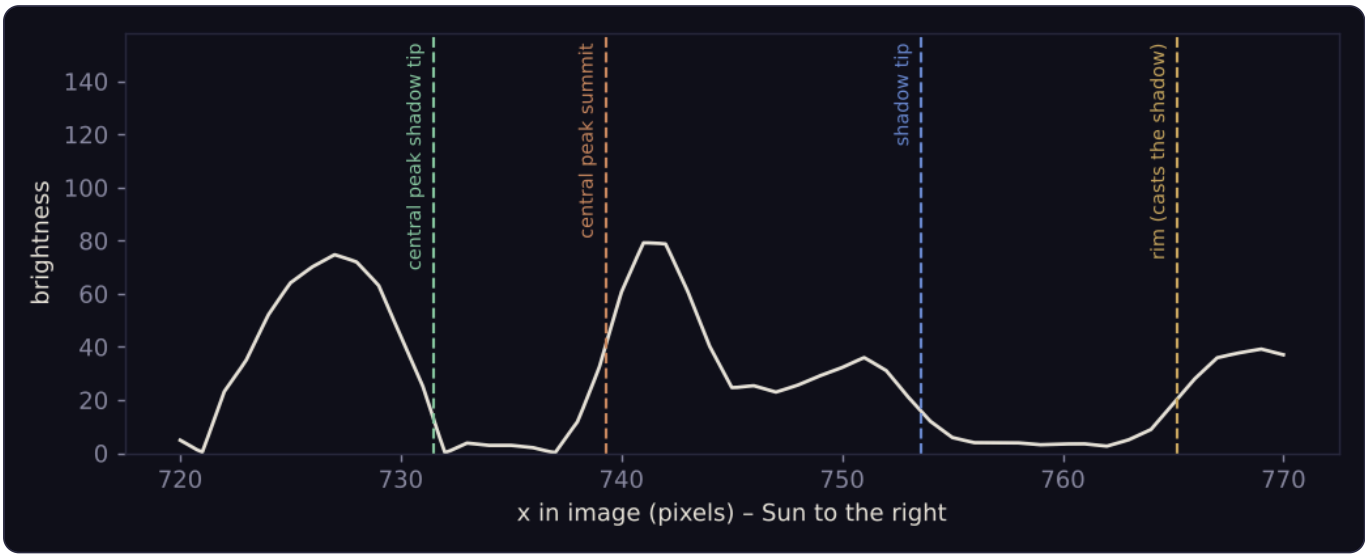
26 March 2026. Sunrise over the Apennines, Archimedes and the Ptolemaeus chain.



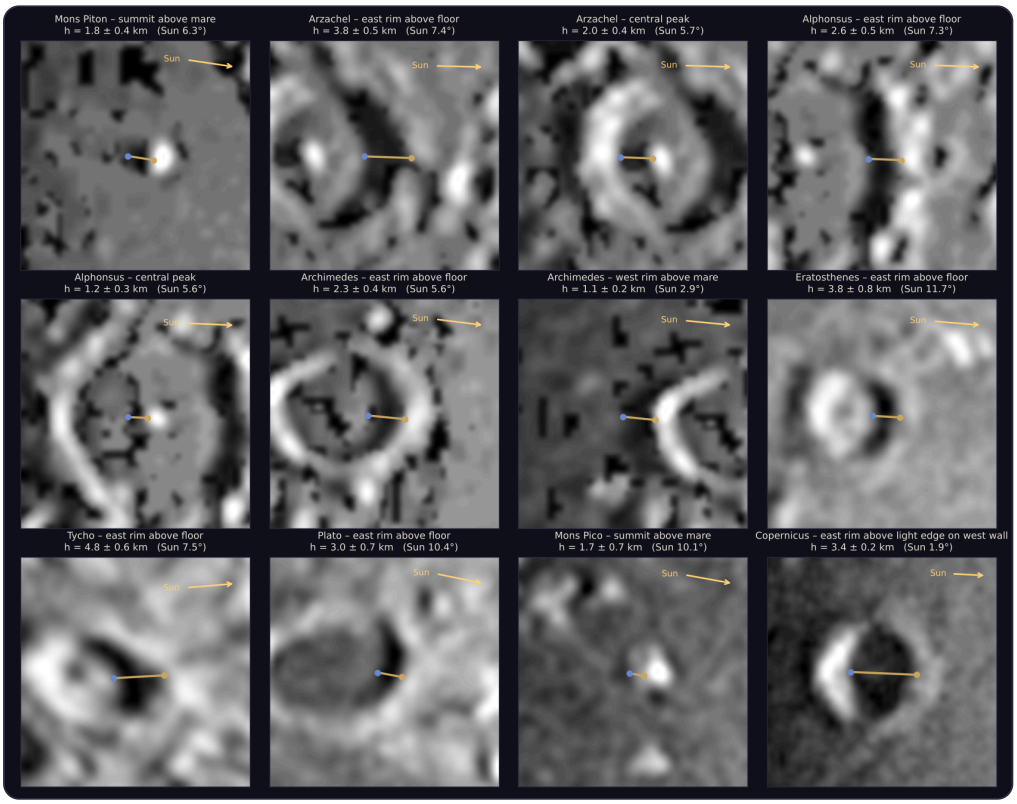
25 May 2026. The terminator runs straight through Copernicus.

HOW A SHADOW IS MEASURED

I lay a line through the feature in the direction of the Sun and read off the brightness along it. The shadow begins where the sunlit slope ends, and it ends where the brightness has risen halfway to the sunlit floor. Both points are converted to selenographic coordinates, and the height follows from the geometry.



Brightness through Arzachel along the direction of the Sun. The east rim casts a shadow 11.6 pixels across the floor, and the central peak one of 7.8 pixels.



All twelve measurements. Gold: the point casting the shadow. Blue: the tip of the shadow. The arrow points towards the Sun.

RESULTS

FEATURE	WHAT WAS MEASURED	SUN ALTITUDE AT TIP	SHADOW	MEASURED HEIGHT	PUBLISHED	LRO
Mons Piton	summit above Mare Imbrium	6.3°	15.5 km	1.8 ± 0.4 km	2.25 km	1.9 km
Arzachel	east rim above floor	7.4°	27.5 km	3.8 ± 0.5 km	depth 3.6 km	4.0 km
Arzachel	central peak	5.7°	18.6 km	2.0 ± 0.4 km	1.5–2.1 km	1.7 km
Alphonsus	east rim above floor	7.3°	19.2 km	2.6 ± 0.5 km	depth 2.7 km	2.4 km
Alphonsus	central peak	5.6°	11.7 km	1.2 ± 0.3 km	1.5–2.2 km	1.1 km
Archimedes	east rim above floor	5.6°	21.8 km	2.3 ± 0.4 km	depth c. 2.1 km	2.0 km
Archimedes	west rim above mare	2.9°	19.1 km	1.1 ± 0.2 km	–	1.1 km
Eratosthenes	east rim above floor	11.7°	17.9 km	3.8 ± 0.8 km	depth 3.4 km	3.1 km
Tycho	east rim above floor	7.5°	33.6 km	4.8 ± 0.6 km	depth 4.7 km	4.7 km
Plato	east rim above floor	10.4°	16.2 km	3.0 ± 0.7 km	1.0–2.0 km	2.0 km
Mons Pico	summit above Mare Imbrium	10.1°	9.6 km	1.7 ± 0.7 km	2.4 km	2.4 km
Copernicus	east rim above the light edge on the west wall	1.9°	65 km	≥ 3.4 ± 0.2 km	depth 3.8 km	3.1 km

The uncertainty corresponds to moving each end of the shadow by one pixel. The crater depths agree with the published values within the uncertainty: Arzachel, Alphonsus, Archimedes, Eratosthenes and Tycho. Copernicus only gives a lower limit, because with the Sun just 1.9° above the horizon the whole floor is in shadow and the light only begins up

on the west wall. On the other hand, the central peaks catch the sunlight in the middle of the dark crater.

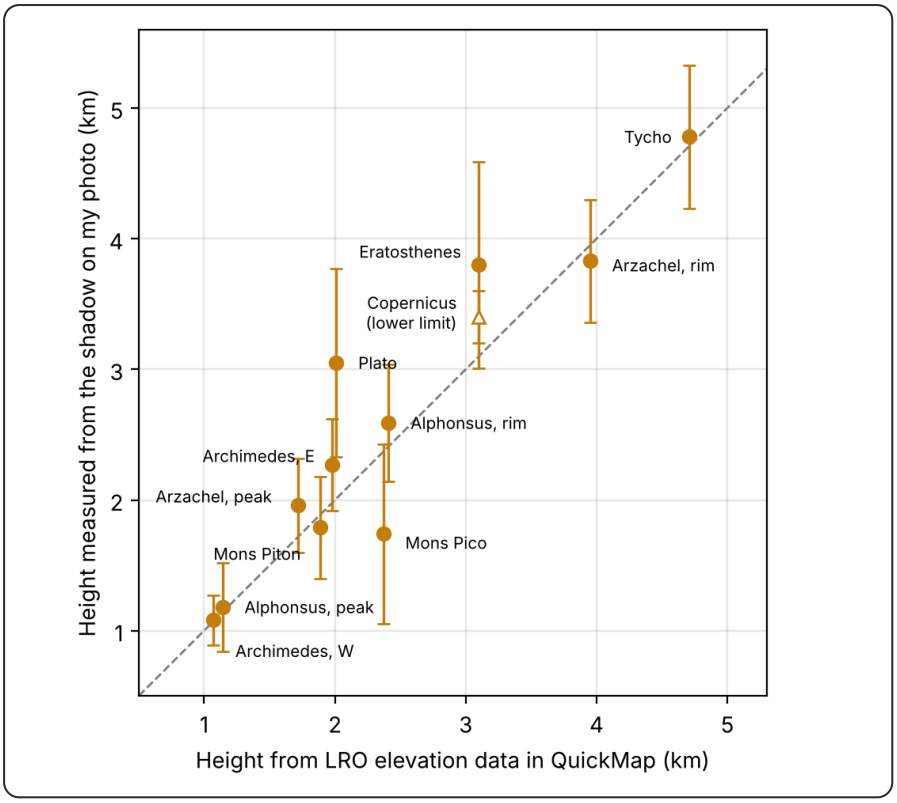
Are the small mountains too low? Mons Piton and the central peak of Alphonsus look too low compared with the published values, but the LRO profiles below show that they agree well when the height is measured from the tip of the shadow. The published values use a different baseline. Only Mons Pico comes out too low. Its shadow is less than four pixels long, and atmospheric turbulence smears the image over 2–3 pixels, so the bright sunlit side "bleeds" into the shadow and shortens it. Long shadows, like those of the craters, are much less affected.

Other sources of error: the images have been sharpened, which moves edges slightly. The time matters: an hour's difference shifts the solar altitude by 0.5° and changes the heights by 5–10 %. The time stamps in the eVscope2 image files are in UT. The Unistellar app shows the same image with local time, and the rotation of the images matches the Moon's parallactic angle at the UT time. An earlier version of this page took the time stamps as local time, which made the heights 4–8 % too low.

A CHECK WITH NASA ELEVATION DATA

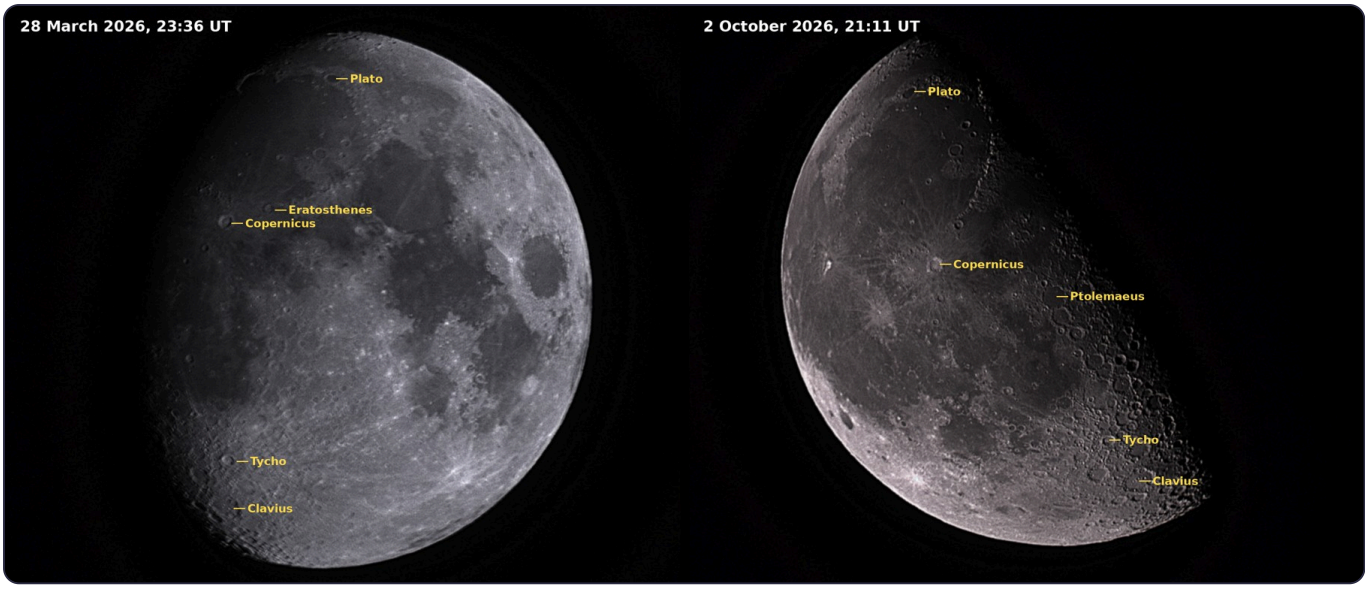
LROC QuickMap is a free map of the Moon that runs directly in the browser, with no installation or login. It is based on data from NASA's Lunar Reconnaissance Orbiter (LRO). Draw a line across a crater, and QuickMap shows the elevation profile along it from the SLDEM2015 elevation model, with a point every 40–80 m. I drew a line along the direction of the Sun through each of my shadows and read the height of the highest point of the rim or peak and the height of the ground where the shadow ends. The difference is exactly the height the shadow method measures; it is listed in the LRO column of the table above. The image calibration can shift points by a couple of pixels along the direction of the Sun, so I used the highest point of the profile and measured the shadow length from there.

Ten of the twelve shadow heights lie within one standard deviation of the LRO value. Plato is 1.4 and Copernicus 1.5 standard deviations too high. The profiles also show that Mons Piton and the central peak of Alphonsus are not too low when measured from the tip of the shadow: LRO gives 1.9 and 1.1 km, just like my shadows. The difference from the published values arises because those are measured from a different baseline. Only Mons Pico, whose shadow is less than four pixels long, comes out too low. For Copernicus the LRO profile gives 3.1 km from the east rim down to the point 65 km further west where my shadow ends. My value of 3.4 km is 1.5 standard deviations higher. The crater's total depth of 3.8 km is larger because the tip of the shadow does not reach the deepest point.



Heights measured with the shadow method on my photos compared with elevation profiles from NASA's LRO data in QuickMap. The error bars correspond to moving each end of the shadow by one pixel. The dashed line shows perfect agreement.

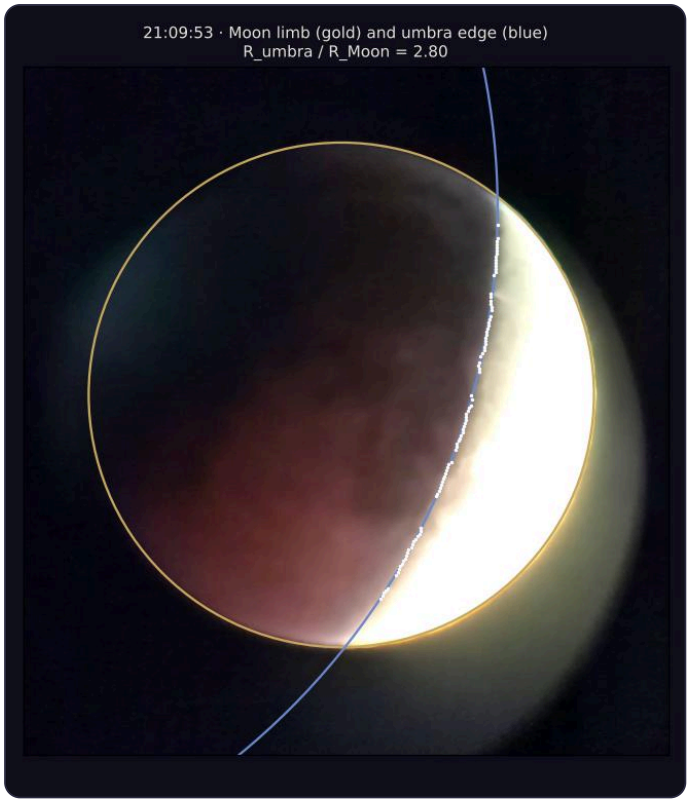
Students without a telescope of their own can do the exercise on other people's images, for example mine on the astrophoto page, where the time is given in UT in the file name. They measure the shadow length in pixels, find the scale of the image from the diameter of the Moon, look up the colongitude in Virtual Moon Atlas and calculate the height. Then they draw the same line in QuickMap and see how close they came to the LRO profile.



Two of my eVscope2 images for students to measure. Left: waxing Moon on 28 March 2026 at 23:36 UT, with Copernicus just at the morning terminator. Right: waning Moon on 2 October 2026 at 21:11 UT, when the Sun is setting, so the shadows fall the other way.

2 · The Earth's shadow and the distance of the Moon

During the total lunar eclipse of 7 September 2025 I photographed the Moon through my Celestron AstroFi 6 with my phone held up to the eyepiece just as it was leaving the Earth's umbra. My eVscope2 could not be used that evening, because the sky was too bright for it to find stars to orient itself by. The edge of the shadow is part of a circle, and its curvature tells how large the Earth's shadow is compared with the Moon.



21:09:53 CEST. One circle is fitted to the limb of the Moon (gold) and another to the edge of the umbra (blue; the white points are the detected edge points).

The radius of the shadow in lunar radii is

$$k = R_{umbra} / R_{Moon} = 2.76 \pm 0.08$$

The value is the mean of nine fits with different smoothing and cut-offs (2.64–2.86). Pure geometry gives 2.70 for that evening, and with the 2 % by which the Earth's atmosphere enlarges the shadow (Danjon's rule) it becomes 2.75.

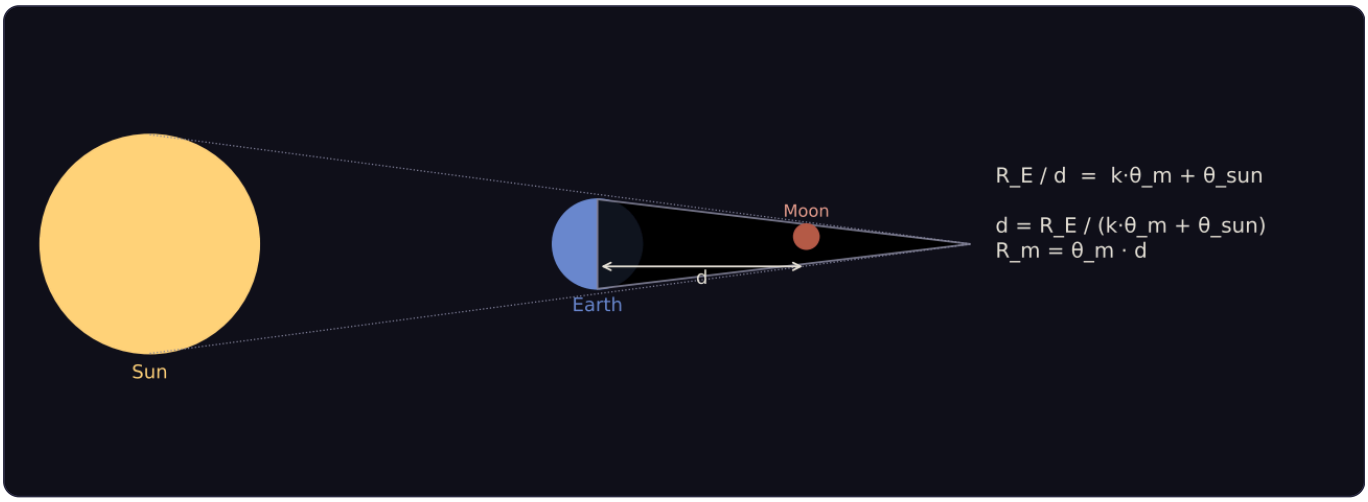


The four images rotated to the same orientation using the maria. s is the distance from the centre of the Moon to the centre of the shadow in lunar radii, found from where the shadow edge crosses the axis and compared with the ephemeris. From 20:53 to 21:10 the shadow moves 0.44 lunar radii (ephemeris 0.48).

THE CALCULATION OF ARISTARCHUS AND HIPPARCHUS

The Earth's shadow is a cone, because the Sun is larger than the Earth. At the Moon's distance d, the angular radius of the cone is the Earth's radius seen from the Moon minus the angular radius of the Sun:

$$R_E / d = k \cdot \theta_m + \theta_{sun}$$



The shadow cone (not to scale).

Here θ_m is the angular radius of the Moon and $\theta_{sun} = 0.265°$ that of the Sun. The Moon's angular diameter that evening can be read off in Stellarium: 32.43′ at 21:07, so $\theta_m = 0.2703°$. With $R_E = 6371$ km this gives d = 361,000 ± 8,000 km. But the measured k includes the 2 % from the atmosphere (Danjon's rule), whereas the formula holds for the geometric shadow. Dividing k by 1.02, the result becomes (the true distance from the centre of the Earth was c. 369,600 km; Stellarium gives 368,478 km, but that is the distance from Djursland)

DISTANCE TO THE MOON

366,500 km

± 8,000 km · true: c. 369,600 km

RADIUS OF THE MOON

1730 km

± 37 km · true value: 1737 km

RADIUS OF THE UMBRA

2.76 R_m

± 0.08 · expected 2.75

Aristarchus could not measure the angular size of the Moon accurately, but he knew that the Sun and the Moon look the same size. If we put $\theta_{sun} = \theta_m$, the angles drop out entirely for the radius of the Moon:

$$R_m = R_E / (k + 1) = 6371\ km / 3.76 \approx 1690\ km$$

This is the result of Aristarchus and Hipparchus in modern form: the Moon is just under a quarter the size of the Earth. With the classical angular radius of 0.25°, the same calculation gives a distance of 383,000 km, close to the mean distance of 384,400 km. My eVscope2 images from other nights give an average $\theta_m = 0.2657°$, but that is not good enough: the Moon's distance varies by ±5 %, and that evening the Moon was close to perigee.

Sources of error. The edge of the umbra is not sharp, because the penumbra and light refracted in the Earth's atmosphere blend into it. I used the steepest rise in brightness, and since the bright part is overexposed, the edge may lie slightly too far out, which makes k up to 0.04 too large. The shadow arc covers only part of the Moon, and it is the curvature of the arc that determines k. That is the largest single uncertainty.

A CHECK WITH STELLARIUM

I compared with screenshots of the same eclipse from the planetarium program Stellarium. Stellarium puts the Earth's shadow in the right place, but draws its edge as a soft transition that curves far too much. Circles fitted to the edge give k ≈ 1.0–1.6, so the curvature cannot be used. The *position* of the edge is right, however. If s is the distance from the centre of the Moon to the centre of the shadow, calculated from the coordinates of the Moon and the Sun, and x is the distance of the edge from the centre of the Moon, counted positive towards the bright side and negative towards the dark side, then

$$k = s + x$$

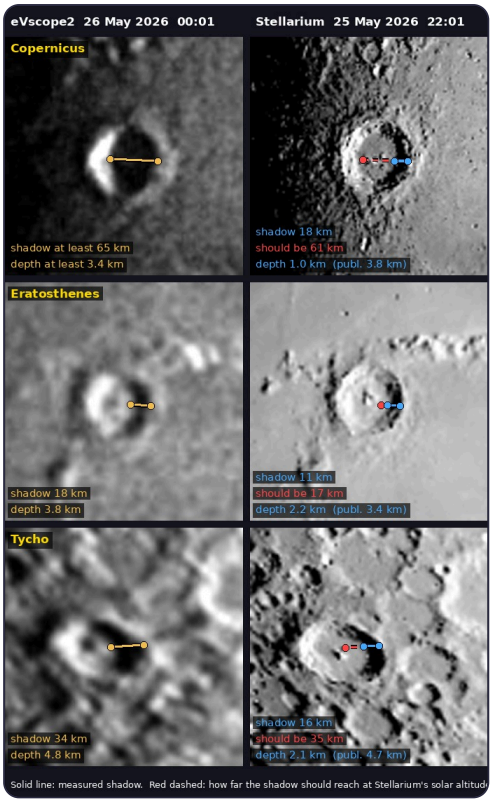
TIME (CEST)	S	X	K
21:07:44	2.17	+0.56	2.74
21:25:44	2.74	−0.03	2.71
21:45:44	3.38	c. −0.6	c. 2.78

This makes a good exercise for students without a telescope: read off RA and Dec for the Moon and the Sun in Stellarium, add 12 hours to the Sun's RA and change the sign of its

Dec to find the centre of the shadow, and calculate the angular distance with the spherical law of cosines. For the shadow heights, however, Stellarium is not reliable. Stellarium on the phone draws the shadows of mountains and craters from elevation data, but the shadows are too short. I compared my eVscope2 image of 26 May 2026 at 00:01 CEST (22:01 UT on 25 May) with Stellarium set to 25 May at 22:01 CEST and measured the shadows in the same way. The Stellarium image is thus 2 hours older than the photo, but each shadow is compared with the solar altitude at its own time. Two hours earlier the Sun stood lower, so Stellarium's shadows should, if anything, have been longer than the photo's:

FEATURE	STELLARIUM	MY PHOTO	PUBLISHED
Copernicus	1.0 km	≥ 3.4 km	3.8 km
Eratosthenes	2.2 km	3.8 km	3.4 km
Tycho	2.1 km	4.8 km	4.7 km
Plato	0.9 km	3.0 km	1.0–2.0 km
Mons Pico	1.5 km	1.7 km	2.4 km

For the craters with a reliable published value, Stellarium gives only 27–66 % of the true height, on average about half. Only for Mons Pico do Stellarium and the photo agree, and there both are too low, because the shadow is under four pixels long in the photo. A likely explanation is that Stellarium computes the shadows from a precomputed elevation map of limited resolution, which rounds off the sharp crater rims. Stellarium 23.1 added an option to sharpen the shadows on the Moon with supersampling.



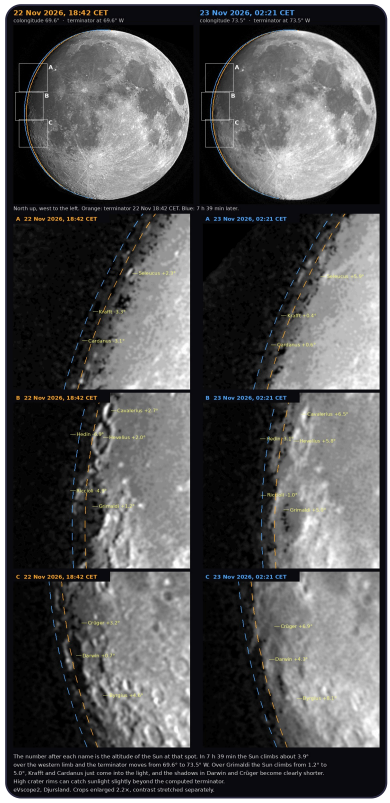
My eVscope2 image of 26 May 2026 at 00:01 CEST (left) and Stellarium on 25 May at 22:01 CEST (right). Solid line: the measured shadow. Red dashed line: how far the shadow should reach at Stellarium's solar altitude, given the known depth of the crater. In the photo the whole floor of Copernicus is in shadow; in Stellarium only a narrow arc.

The time of the comparison was set by my photo. On 25 May the Moon was just past first quarter, and Copernicus, Eratosthenes and Tycho lay near the terminator, where the shadows are long. That the sky is bright on a summer evening hardly matters, because the Moon is far brighter than the twilight sky. What counts is the altitude of the Sun at the crater on the Moon, not how dark it is here. In Stellarium the atmosphere can be switched off (key A), making the sky black even in twilight. For teaching, autumn and winter evenings are still the best: the first-quarter Moon then stands high in a dark sky early in the evening.

3 · The wandering terminator

The terminator is the boundary between day and night on the Moon. It moves about 0.5° in selenographic longitude per hour, because the Moon turns once relative to the Sun in 29.5 days. This can be seen in two images taken a few hours apart. My two eVscope2 images were taken on 22 November 2026 at 18:42 CET and on 23 November at 02:21, 7 h 39 min later. The Moon was 12 days old, so the terminator lay right out at the western limb. The colongitude rose from 69.6° to 73.5°, and the terminator moved correspondingly from 69.6° to 73.5° west.

On the full disk the shift is only a narrow strip at the limb, so the figure also shows three enlarged crops. The number after each crater is the altitude of the Sun above the horizon at that spot, computed from the positions of the Moon and the Sun. Krafft and Cardanus go from night (−3.3° and −3.1°) to sunrise. Over Grimaldi the Sun climbs from 1.2° to 5.0°, and over Darwin from 0.7° to 4.3°. The shadows behind the crater rims become clearly shorter. High rims can catch sunlight slightly beyond the computed terminator, just as mountain tops on Earth are sunlit before the Sun rises in the valley.



The terminator on 22 November 2026 at 18:42 CET (orange) and 7 h 39 min later (blue). The images are rotated so the Moon's north pole is up. The crops are enlarged 2.2 times and the contrast is stretched separately, so the brightness cannot be compared directly.

This makes a good exercise for students: take two images on the same night, look up the colongitude for both times in Stellarium or Virtual Moon Atlas, and see which craters have come into the light. With the shadow method of section 1 one can also see that the shadows get shorter as the Sun rises.

4 · Be careful with Stellarium

Stellarium is an excellent program for finding out where and when something can be seen, and its coordinates are accurate. But the picture on the screen is a drawing, not a photograph, and that must be remembered when students want to measure on it:

- **The shadows are too short.** Stellarium draws the Moon's shadows from an elevation map of limited resolution. Compared with my photo of 25–26 May, the shadows of Copernicus, Eratosthenes and Tycho were on average only about half as long as they should be (section 2). Heights measured in Stellarium therefore come out too small.
- **The edge of the Earth's shadow is curved wrongly.** During the eclipse the edge of the shadow is in the right place, but it is drawn as a soft transition that curves far too much. Circles fitted to the edge give $k \approx 1.0$–1.6 instead of about 2.7. Use the position of the edge and Stellarium's coordinates, not the curvature of the edge.
- **The distance is from your own location.** Stellarium shows the distance from the observing site (topocentric), not from the centre of the Earth. That evening the difference was about 1,100 km. The formulas of Aristarchus' method give the distance from the centre of the Earth.
- **Check time and place.** Stellarium uses the time zone and location that have been set. An hour's error moves the terminator about 0.5° and changes the solar altitude accordingly. Check the time stamps of your own images too: the eVscope2 stores them in UT, which I only discovered when a Stellarium screenshot at 18:42 CET matched a photo stamped 17:42 exactly.
- **The brightness of the sky is simulated.** In twilight the sky looks bright and washed out, as it does in reality. Switch off the atmosphere (key A) to see the Moon clearly.
- **Phone and computer may draw differently.** The phone and desktop versions do not always have the same settings and textures. My comparisons were made with Stellarium on a phone.

In short: use Stellarium for numbers (coordinates, times, angular sizes, colongitude) and real photographs for measuring shadows and shapes.

Michael H. Grant · lektor emeritus in physics and mathematics · November 2026
Sources of published values: The Moon Wiki (the-moon.us), Wikipedia, a Cambridge Moon atlas, and Margot et al. (1999) for Tycho. Calculations: my own Python programs with astropy and the IAU 2009 model for the Moon's rotation.